Specialist Mathematics Unit 2: **Chapter 9**

Ex 9A

1.
L.H.S.:

$$\begin{aligned} 2\cos^2\theta + 3 &= 2(1-\sin^2\theta)+3 \\ &= 2 - 2\sin^2\theta + 3 \\ &= 5 - 2\sin^2\theta \\ &= \text{R.H.S.} \end{aligned}$$

2.
L.H.S.:

$$\begin{aligned} \sin\theta - \cos^2\theta &= \sin\theta - (1-\sin^2\theta) \\ &= \sin\theta - 1 + \sin^2\theta \\ &= \sin\theta + \sin^2\theta - 1 \\ &= (\sin\theta)(1+\sin\theta) - 1 \\ &= \text{R.H.S.} \end{aligned}$$

3.
L.H.S.:

$$\begin{aligned} (\sin\theta + \cos\theta)^2 &= \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta \\ &= 2\sin\theta\cos\theta + \sin^2\theta + \cos^2\theta \\ &= 2\sin\theta\cos\theta + 1 \\ &= \text{R.H.S.} \end{aligned}$$

4.
R.H.S.:

$$\begin{aligned} (\sin\theta - \cos\theta)^2 &= \sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta \\ &= \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta \\ &= 1 - 2\sin\theta\cos\theta \\ &= \text{L.H.S.} \end{aligned}$$

5.
L.H.S.:

$$\sin^4\theta - \cos^4\theta = (\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)$$

(difference of perfect squares)

$$\begin{aligned} &= 1(\sin^2\theta - \cos^2\theta) \\ &= (1-\cos^2\theta) - \cos^2\theta \\ &= 1 - 2\cos^2\theta \\ &= \text{R.H.S.} \end{aligned}$$

6.
L.H.S.:

$$\begin{aligned} \sin^4\theta - \sin^2\theta &= \sin^2\theta(\sin^2\theta - 1) \\ &= (1-\cos^2\theta)(-\cos^2\theta) \\ &= -\cos^2\theta + \cos^4\theta \\ &= \cos^4\theta - \cos^2\theta \\ &= \text{R.H.S.} \end{aligned}$$

7.
L.H.S.:

$$\begin{aligned} \sin^2\theta\tan^2\theta &= (1-\cos^2\theta)\tan^2\theta \\ &= \tan^2\theta - \cos^2\theta\tan^2\theta \\ &= \tan^2\theta - \cos^2\theta\frac{\sin^2\theta}{\cos^2\theta} \\ &= \tan^2\theta - \sin^2\theta \\ &= \text{R.H.S.} \end{aligned}$$

8.
L.H.S.:

$$\begin{aligned} (1+\sin\theta)(1-\sin\theta) &= 1 - \sin^2\theta \\ &= \cos^2\theta \\ &= 1 + \cos^2\theta - 1 \\ &= 1 + (\cos\theta + 1)(\cos\theta - 1) \\ &= \text{R.H.S.} \end{aligned}$$

9.
L.H.S.:

$$\begin{aligned} \sin\theta\tan\theta + \cos\theta &= \sin\theta\frac{\sin\theta}{\cos\theta} + \cos\theta \\ &= \frac{\sin^2\theta}{\cos\theta} + \cos\theta \\ &= \frac{1-\cos^2\theta}{\cos\theta} + \cos\theta \\ &= \frac{1}{\cos\theta} - \frac{\cos^2\theta}{\cos\theta} + \cos\theta \\ &= \frac{1}{\cos\theta} - \cos\theta + \cos\theta \\ &= \frac{1}{\cos\theta} \\ &= \text{R.H.S.} \end{aligned}$$

10.

L.H.S.:

$$\frac{1}{1+\tan^2\theta} = \frac{1}{1+\frac{\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{1}{\frac{\cos^2\theta}{\cos^2\theta}+\frac{\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{1}{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{\cos^2\theta}{\cos^2\theta+\sin^2\theta}$$

$$= \frac{\cos^2\theta}{1}$$

$$= \cos^2\theta$$

$$= \text{R.H.S.}$$

11.

R.H.S.:

$$\frac{1+\cos\theta}{1-\cos\theta} = \frac{1+\cos\theta}{1-\cos\theta}\times\frac{1+\cos\theta}{1+\cos\theta}$$

$$= \frac{1+2\cos\theta+\cos^2\theta}{1-\cos^2\theta}$$

$$= \frac{\cos^2\theta+2\cos\theta+1}{\sin^2\theta}$$

$$= \text{L.H.S.}$$

12.

L.H.S.:

$$\frac{\sin\theta}{1-\cos\theta}-\frac{\cos\theta}{\sin\theta} = \frac{\sin\theta}{1-\cos\theta}\times\frac{1+\cos\theta}{1+\cos\theta}-\frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}-\frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin\theta(1+\cos\theta)}{\sin^2\theta}-\frac{\cos\theta}{\sin\theta}$$

$$= \frac{1+\cos\theta}{\sin\theta}-\frac{\cos\theta}{\sin\theta}$$

$$= \frac{1+\cos\theta-\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta}$$

$$= \text{R.H.S.}$$

13.

L.H.S.:

$$\frac{1-\sin\theta\cos\theta-\cos^2\theta}{\sin^2\theta+\sin\theta\cos\theta-1} = \frac{1-\sin\theta\cos\theta-(1-\sin^2\theta)}{(1-\cos^2\theta)+\sin\theta\cos\theta-1}$$

$$= \frac{1-\sin\theta\cos\theta-1+\sin^2\theta}{1-\cos^2\theta+\sin\theta\cos\theta-1}$$

$$= \frac{-\sin\theta\cos\theta+\sin^2\theta}{-\cos^2\theta+\sin\theta\cos\theta}$$

$$= \frac{\sin\theta(-\cos\theta+\sin\theta)}{\cos\theta(-\cos^2\theta+\sin\theta)}$$

$$= \frac{\sin\theta}{\cos\theta}$$

$$= \tan\theta$$

$$= \text{R.H.S.}$$

Ex 9B

Note Ques 1 to 3 are slightly different from your text book….but you can get the general idea

1.

To prove: $\sin(x+2\pi)=\sin x$

Proof:

$$\begin{aligned}\text{L.H.S.} &= \sin(x+2\pi)\\ &= \sin x\cos 2\pi+\cos x\sin 2\pi\\ &= \sin x\times 1+\cos x\times 0\\ &= \sin x\\ &= \text{R.H.S}\end{aligned}$$

2.

To prove: $\cos(x+2\pi)=\cos x$

Proof:

$$\begin{aligned}\text{L.H.S.} &= \cos(x+2\pi)\\ &= \cos x\cos 2\pi-\sin x\sin 2\pi\\ &= \cos x\times 1+\sin x\times 0\\ &= \cos x\\ &= \text{R.H.S}\end{aligned}$$

3.

To prove: $\sin(x-2\pi)=\sin x$

Proof:

$$\begin{aligned}\text{L.H.S.} &= \sin(x-2\pi)\\ &= \sin x\cos 2\pi-\cos x\sin 2\pi\\ &= \sin x\times 1-\cos x\times 0\\ &= \sin x\\ &= \text{R.H.S}\end{aligned}$$

5.

To prove: $\sin(\mathrm{A}+\mathrm{B})-\sin(\mathrm{A}-\mathrm{B})=2\cos\mathrm{A}\sin\mathrm{B}$

Proof:

$$\begin{aligned}\text{L.H.S.} &= \sin(\mathrm{A}+\mathrm{B})-\sin(\mathrm{A}-\mathrm{B})\\ &= \sin\mathrm{A}\cos\mathrm{B}+\cos\mathrm{A}\sin\mathrm{B}\\ &\qquad -(\sin\mathrm{A}\cos\mathrm{B}-\cos\mathrm{A}\sin\mathrm{B})\\ &= \sin\mathrm{A}\cos\mathrm{B}+\cos\mathrm{A}\sin\mathrm{B}\\ &\qquad -\sin\mathrm{A}\cos\mathrm{B}+\cos\mathrm{A}\sin\mathrm{B}\\ &= 2\cos\mathrm{A}\sin\mathrm{B}\\ &= \text{R.H.S}\end{aligned}$$

6.

To prove: $\cos(\mathrm{A}-\mathrm{B})+\cos(\mathrm{A}+\mathrm{B})=2\cos\mathrm{A}\cos\mathrm{B}$

Proof:

$$\begin{aligned}\text{L.H.S.} &= \cos(\mathrm{A}-\mathrm{B})+\sin(\mathrm{A}+\mathrm{B})\\ &= \cos\mathrm{A}\cos\mathrm{B}+\sin\mathrm{A}\sin\mathrm{B}\\ &\qquad +\cos\mathrm{A}\cos\mathrm{B}-\sin\mathrm{A}\sin\mathrm{B}\\ &= 2\cos\mathrm{A}\cos\mathrm{B}\\ &= \text{R.H.S}\end{aligned}$$

7.

To prove: $2\cos\left(x-\frac{\pi}{6}\right)=\sin x+\sqrt{3}\cos x$

Proof:

$$\begin{aligned}\text{L.H.S.} &= 2\cos\left(x-\frac{\pi}{6}\right)\\ &= 2\left(\cos x\cos\frac{\pi}{6}+\sin x\sin\frac{\pi}{6}\right)\\ &= 2\left(\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x\right)\\ &= \sqrt{3}\cos x+\sin x\\ &= \sin x+\sqrt{3}\cos x\\ &= \text{R.H.S}\end{aligned}$$

8.

To prove: $\tan\left(\theta+\frac{\pi}{4}\right)=\frac{1+\tan\theta}{1-\tan\theta}$

Proof:

$$\begin{aligned}\text{L.H.S.} &= \tan\left(\theta+\frac{\pi}{4}\right)\\ &= \frac{\tan\theta+\tan\frac{\pi}{4}}{1-\tan\theta\tan\frac{\pi}{4}}\\ &= \frac{\tan\theta+1}{1-\tan\theta\times 1}\\ &= \frac{1+\tan\theta}{1-\tan\theta}\\ &= \text{R.H.S}\end{aligned}$$

9.

To prove: $\frac{\cos(A+B)}{\cos(A-B)} = \frac{1-\tan A \tan B}{1+\tan A \tan B}$

Proof:

$$\begin{aligned}
\text{L.H.S.} &= \frac{\cos(A+B)}{\cos(A-B)} \\
&= \frac{\cos A \cos B - \sin A \sin B}{\cos A \cos B + \sin A \sin B} \\
&= \frac{\frac{\cos A \cos B - \sin A \sin B}{\cos A \cos B}}{\frac{\cos A \cos B + \sin A \sin B}{\cos A \cos B}} \\
&= \frac{1 - \frac{\sin A \sin B}{\cos A \cos B}}{1 + \frac{\sin A \sin B}{\cos A \cos B}} \\
&= \frac{1 - \frac{\sin A}{\cos A} \times \frac{\sin B}{\cos B}}{1 + \frac{\sin A}{\cos A} \times \frac{\sin B}{\cos B}} \\
&= \frac{1 - \tan A \tan B}{1 + \tan A \tan B} \\
&= \text{R.H.S}
\end{aligned}$$

10.

To prove:

$$\sqrt{2}(\sin x - \cos x)\sin(x+45^\circ) = 1 - 2\cos^2 x$$

Proof:

$$\begin{aligned}
\text{L.H.S.} &= \sqrt{2}(\sin x - \cos x)\sin(x+45^\circ) \\
&= \sqrt{2}(\sin x - \cos x)\,(\sin x \cos 45^\circ \\
&\qquad + \cos x \sin 45^\circ) \\
&= \sqrt{2}(\sin x - \cos x)\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right) \\
&= (\sin x - \cos x)(\sin x + \cos x) \\
&= \sin^2 x - \cos^2 x \\
&= (1 - \cos^2 x) - \cos^2 x \\
&= 1 - 2\cos^2 x \\
&= \text{R.H.S}
\end{aligned}$$

11.

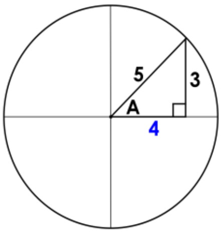
To prove: $\tan\left(\theta + \frac{\pi}{4}\right) = \frac{1+2\sin\theta\cos\theta}{1-2\sin^2\theta}$

Proof:

$$\begin{aligned}
\text{L.H.S.} &= \tan\left(\theta + \frac{\pi}{4}\right) \\
&= \frac{\tan\theta + \tan\frac{\pi}{4}}{1 - \tan\theta \tan \pi 4} \\
&= \frac{\tan\theta + 1}{1 - \tan\theta \times 1} \\
&= \frac{\frac{\sin\theta}{\cos\theta} + 1}{1 - \frac{\sin\theta}{\cos\theta}} \\
&= \frac{\frac{\sin\theta + \cos\theta}{\cos\theta}}{\frac{\cos\theta - \sin\theta}{\cos\theta}} \\
&= \frac{\sin\theta + \cos\theta}{\cos\theta - \sin\theta} \\
&= \frac{\sin\theta + \cos\theta}{\cos\theta - \sin\theta} \times \frac{\sin\theta + \cos\theta}{\cos\theta + \sin\theta} \\
&= \frac{\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta}{\cos^2\theta - \sin^2\theta} \\
&= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta}{1 - \sin^2\theta - \sin^2\theta} \\
&= \frac{1 + 2\sin\theta\cos\theta}{1 - 2\sin^2\theta} \\
&= \text{R.H.S}
\end{aligned}$$

Ex 9C

1.



(a) $\sin 2A = 2\sin A\cos A$

$= 2 \times \frac{3}{5} \times -\frac{4}{5}$

$= -\frac{24}{25}$

(b) $\cos 2A = 2\cos^2 A - 1$

$= 2 \times \left(\frac{4}{5}\right)^2 - 1$

$= \frac{7}{25}$

(c) $\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$

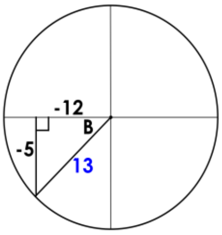
$= \frac{2 \times -\frac{3}{4}}{1 - (-\frac{3}{4})^2}$

$= \frac{-\frac{6}{4}}{\frac{7}{16}}$

$= -\frac{3}{2} \times \frac{16}{7}$

$= -\frac{24}{7}$

2.



(a) $\sin 2B = 2\sin B\cos B$

$= 2(-\frac{5}{13})(-\frac{12}{13})$

$= \frac{120}{169}$

(b) $\cos 2B = 2\cos^2 B - 1$

$= 2(-\frac{12}{13})^2 - 1$

$= \frac{288 - 169}{169}$

$= \frac{119}{169}$

(c) $\tan 2B = \frac{\sin 2B}{\cos 2B}$

$= \frac{120}{119}$

3.

(a) $6\sin A\cos A = 3(2\sin A\cos A) = 3\sin 2A$

(b) $4\sin 2A\cos 2A = 2(2\sin 2A\cos 2A)$

$= 2\sin(2 \times 2A)$

$= 2\sin 4A$

(c) $\sin\frac{A}{2}\cos\frac{A}{2} = \frac{1}{2}(2\sin\frac{A}{2}\cos\frac{A}{2})$

$= \frac{1}{2}\sin(2 \times \frac{A}{2})$

$= \frac{1}{2}\sin A$

4.

(a) $2\cos^2 2A - 2\sin^2 2A = 2(\cos^2 2A - \sin^2 2A)$

$= 2\cos(2 \times 2A)$

$= 2\cos 4A$

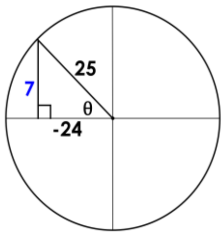
(b) $1 - 2\sin^2\frac{A}{2} = \cos(2 \times \frac{A}{2})$

$= \cos A$

(c) $2\cos^2 2A - 1 = \cos(2 \times 2A)$

$= \cos 4A$

5.



(a) $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2 \times \frac{7}{25} \times -\frac{24}{25}$

$= -\frac{336}{625}$

(b) $\cos 2\theta = 2\cos^2\theta - 1$

$= 2 \times \left(\frac{24}{25}\right)^2 - 1$

$= \frac{1152 - 625}{625}$

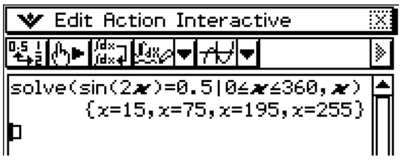
$= \frac{527}{625}$

(c) $\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta}$

$= -\frac{336}{527}$

6.

$$\begin{aligned}4\sin x\cos x &= 1\\ 2(2\sin x\cos x) &= 1\\ 2\sin 2x &= 1\\ \sin 2x &= \frac{1}{2}\end{aligned}$$



This will have four solutions with $2x$ in 1st and 2nd quadrant.

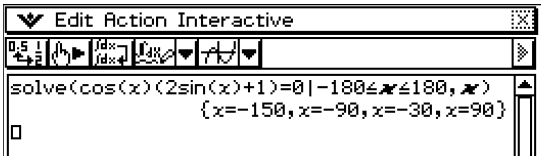
$$\begin{aligned}2x &= 30^\circ & \text{or}\quad 2x &= 180^\circ - 30^\circ\\ x &= 15^\circ & 2x &= 150^\circ\\ & & x &= 75^\circ\end{aligned}$$

$$\begin{aligned}2x &= 360^\circ + 30^\circ & \text{or}\quad 2x &= 540^\circ - 30^\circ\\ 2x &= 390^\circ & 2x &= 510^\circ\\ x &= 195 & x &= 255^\circ\end{aligned}$$

7.

$$\begin{aligned}\sin 2x + \cos x &= 0\\ 2\sin x\cos x + \cos x &= 0\\ \cos x(2\sin x + 1) &= 0\end{aligned}$$

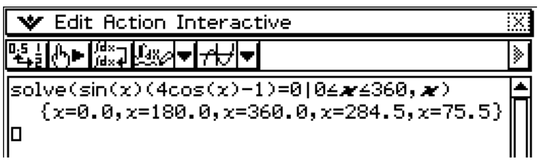
$$\begin{aligned}\cos x &= 0 & \text{or}\quad 2\sin x + 1 &= 0\\ x &= \pm 90^\circ & 2\sin x &= -1\\ & & \sin x &= -\frac{1}{2}\\ & & x &= -30^\circ\\ & & \text{or } x &= -180^\circ + 30^\circ\\ & & &= -150^\circ\end{aligned}$$



8.

$$\begin{aligned}2\sin 2x - \sin x &= 0\\ 4\sin x\cos x - \sin x &= 0\\ \sin x(4\cos x - 1) &= 0\end{aligned}$$

$$\begin{aligned}\sin x &= 0 & \text{or}\quad 4\cos x - 1 &= 0\\ x &= 0^\circ & 4\cos x &= 1\\ \text{or } x &= 180^\circ & \cos x &= \frac{1}{4}\\ \text{or } x &= 360^\circ & x &= 75.5^\circ\\ & & \text{or } x &= 360^\circ - 75.5^\circ\\ & & &= 284.5^\circ\end{aligned}$$

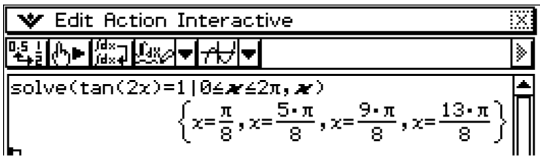


9.

$$\begin{aligned}2\sin x\cos x &= \cos 2x\\ \sin 2x &= \cos 2x\\ \tan 2x &= 1\end{aligned}$$

$$\begin{aligned}2x &= \frac{\pi}{4} & \text{or}\quad 2x &= \pi + \frac{\pi}{4}\\ x &= \frac{\pi}{8} & &= \frac{5\pi}{4}\\ & & x &= \frac{5\pi}{8}\end{aligned}$$

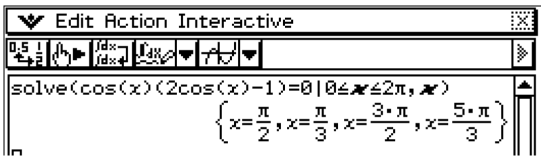
$$\begin{aligned}\text{or}\quad 2x &= 2\pi + \frac{\pi}{4} & \text{or}\quad 2x &= 3\pi + \frac{\pi}{4}\\ &= \frac{9\pi}{4} & &= \frac{13\pi}{4}\\ x &= \frac{9\pi}{8} & x &= \frac{13\pi}{8}\end{aligned}$$



10.

$$\begin{aligned}\cos 2x + 1 - \cos x &= 0\\ 2\cos^2 x - 1 + 1 - \cos x &= 0\\ 2\cos^2 x - \cos x &= 0\\ \cos x(2\cos x - 1) &= 0\end{aligned}$$

$$\begin{aligned}\cos x &= 0 & \text{or}\quad 2\cos x - 1 &= 0\\ x &= \frac{\pi}{2} & 2\cos x &= 1\\ \text{or } x &= \frac{3\pi}{2} & \cos x &= \frac{1}{2}\\ & & x &= \frac{\pi}{3}\\ & & \text{or } x &= 2\pi - \frac{\pi}{3}\\ & & &= \frac{5\pi}{3}\end{aligned}$$



11.

$$\begin{aligned}\cos 2x+\sin x&=0\\ 1-2\sin^2 x+\sin x&=0\\ 2\sin^2 x-\sin x-1&=0\\ (2\sin x+1)(\sin x-1)&=0\end{aligned}$$

$$2\sin x+1=0 \quad \text{or} \quad \sin x-1=0$$

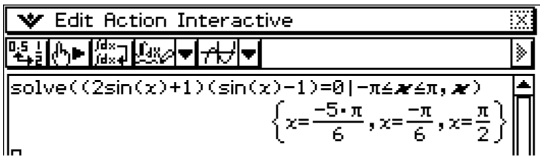
$$2\sin x=-1 \qquad \sin x=1$$

$$\sin x=-\frac{1}{2} \qquad x=\frac{\pi}{4}$$

$$x=-\frac{\pi}{6}$$

$$\text{or}\quad x=-\pi+\frac{\pi}{6}$$

$$=-\frac{5\pi}{6}$$



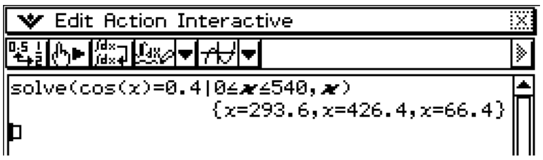
12.

$$\begin{aligned}2\sin^2 x+5\cos x+\cos 2x&=3\\ 2(1-\cos^2 x)+5\cos x+(2\cos^2 x-1)&=3\\ 2-2\cos^2 x+5\cos x+2\cos^2 x-1&=3\\ 1+5\cos x&=3\\ 5\cos x&=2\\ \cos x&=0.4\end{aligned}$$

$$x=66.4^\circ$$

$$x=360-66.4=293.6^\circ$$

$$x=360+66.4=426.4^\circ$$



13.

L.H.S.:

$$\begin{aligned}\sin 2\theta\tan\theta&=2\sin\theta\cos\theta\times\frac{\sin\theta}{\cos\theta}\\ &=2\sin^2\theta\\ &=\text{R.H.S.}\end{aligned}$$

14.

L.H.S.:

$$\begin{aligned}\cos\theta\sin 2\theta&=\cos\theta\times 2\sin\theta\cos\theta\\ &=2\sin\theta\cos^2\theta\\ &=2\sin\theta(1-\sin^2\theta)\\ &=2\sin\theta-2\sin^3\theta)\\ &=\text{R.H.S.}\end{aligned}$$

15.

L.H.S.:

$$\begin{aligned}\frac{1-\cos 2\theta}{1+\cos 2\theta}&=\frac{1-(1-2\sin^2\theta)}{1+(2\cos^2\theta-1)}\\ &=\frac{1-1+2\sin^2\theta}{1+2\cos^2\theta-1}\\ &=\frac{2\sin^2\theta}{2\cos^2\theta}\\ &=\frac{\sin^2\theta}{\cos^2\theta}\\ &=\tan^2\theta\\ &=\text{R.H.S.}\end{aligned}$$

16.

L.H.S.:

$$\begin{aligned}\sin\theta\tan\frac{\theta}{2}&=2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\times\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\\ &=2\sin^2\frac{\theta}{2}\\ &=2(1-\cos^2\frac{\theta}{2})\\ &=2-2\cos^2\frac{\theta}{2}\\ &=\text{R.H.S.}\end{aligned}$$

17.

$$\begin{aligned}\sin 4\theta&=2\sin 2\theta\cos 2\theta\\ &=2(2\sin\theta\cos\theta)(\cos^2\theta-\sin^2\theta)\\ &=(4\sin\theta\cos\theta)(\cos^2\theta-\sin^2\theta)\\ &=4\sin\theta\cos^3\theta-4\sin^3\theta\cos\theta\\ &=\text{R.H.S.}\end{aligned}$$

18.

L.H.S.:

$$\begin{aligned}\frac{\sin 2\theta - \sin\theta}{1-\cos\theta+\cos 2\theta} &= \frac{2\sin\theta\cos\theta - \sin\theta}{1-\cos\theta+(2\cos^2\theta-1)} \\ &= \frac{\sin\theta(2\cos\theta-1)}{2\cos^2\theta-\cos\theta} \\ &= \frac{\sin\theta(2\cos\theta-1)}{\cos\theta(2\cos\theta-1)} \\ &= \frac{\sin\theta}{\cos\theta} \\ &= \tan\theta \\ &= \text{R.H.S.}\end{aligned}$$

19.

L.H.S.:

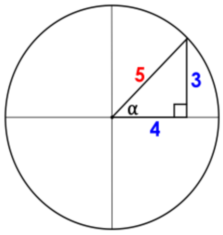
$$\begin{aligned}\cos 4\theta &= 2\cos^2 2\theta - 1 \\ &= 2(\cos 2\theta)^2 - 1 \\ &= 2(2\cos^2\theta - 1)^2 - 1 \\ &= 2(2\cos^2\theta-1)(2\cos^2\theta-1) - 1 \\ &= 2(4\cos^4\theta - 4\cos^2\theta + 1) - 1 \\ &= 8\cos^4\theta - 8\cos^2\theta + 2 - 1 \\ &= 8\cos^4\theta - 8\cos^2\theta + 1 \\ &= 1 - 8\cos^2\theta + 8\cos^4\theta \\ &= \text{R.H.S.}\end{aligned}$$

Ex 9D

1.

$$a\cos(\theta+\alpha) = a(\cos\theta\cos\alpha - \sin\theta\sin\alpha)$$

$$\sqrt{3^2+4^2} = 5$$



$$\alpha = \cos^{-1}\frac{3}{5} = 53.1°$$

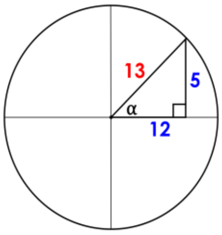
hence

$$3\cos\theta - 4\sin\theta = 5\cos(\theta + 53.1°)$$

2.

$$a\cos(\theta+\alpha) = a(\cos\theta\cos\alpha - \sin\theta\sin\alpha)$$

$$\sqrt{12^2+5^2} = 13$$



$$\alpha = \cos^{-1}\frac{12}{13} = 22.6°$$

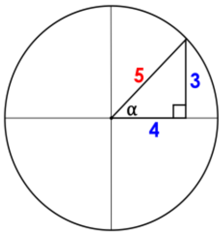
hence

$$12\cos\theta - 5\sin\theta = 13\cos(\theta + 22.6°)$$

3.

$$a\cos(\theta-\alpha) = a(\cos\theta\cos\alpha + \sin\theta\sin\alpha)$$

$$\sqrt{4^2+3^2} = 5$$



$$\alpha = \cos^{-1}\frac{4}{5} = 0.64$$

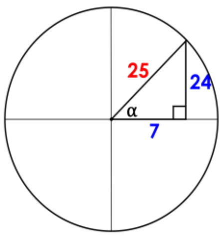
hence

$$4\cos\theta + 3\sin\theta = 5\cos(\theta - 0.64)$$

4.

$$a\cos(\theta-\alpha) = a(\cos\theta\cos\alpha + \sin\theta\sin\alpha)$$

$$\sqrt{7^2+24^2} = 25$$



$$\alpha = \cos^{-1}\frac{7}{25} = 1.29$$

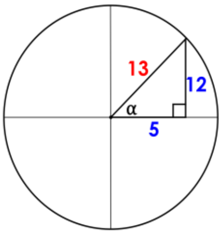
hence

$$7\cos\theta + 24\sin\theta = 25\cos(\theta - 1.29)$$

5.

$$a\sin(\theta+\alpha) = a(\sin\theta\cos\alpha + \cos\theta\sin\alpha)$$

$$\sqrt{5^2+12^2} = 13$$



$$\alpha = \cos^{-1}\frac{5}{13} = 67.4°$$

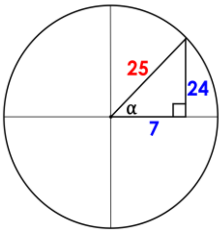
hence

$$5\sin\theta + 12\cos\theta = 13\sin(\theta + 67.4°)$$

6.

$$a\sin(\theta+\alpha) = a(\sin\theta\cos\alpha + \cos\theta\sin\alpha)$$

$$\sqrt{7^2+24^2} = 25$$



$$\alpha = \cos^{-1}\frac{7}{25} = 73.7°$$

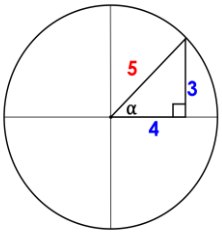
hence

$$7\sin\theta + 24\cos\theta = 25\sin(\theta + 73.7°)$$

7.

$$a\sin(\theta-\alpha) = a(\sin\theta\cos\alpha - \cos\theta\sin\alpha)$$

$$\sqrt{4^2+3^2} = 5$$



$$\alpha = \cos^{-1}\frac{4}{5} = 0.64$$

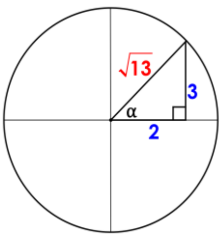
hence

$$4\sin\theta - 3\cos\theta = 5\sin(\theta - 0.64)$$

8.

$$a\sin(\theta-\alpha) = a(\sin\theta\cos\alpha - \cos\theta\sin\alpha)$$

$$\sqrt{2^2+3^2} = \sqrt{13}$$



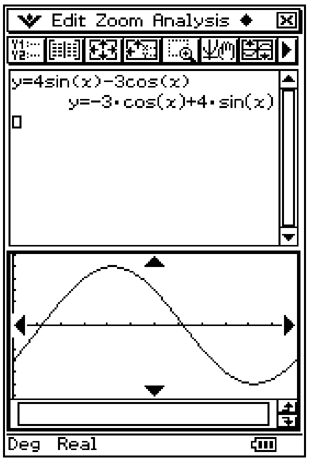
$$\alpha = \cos^{-1}\frac{2}{\sqrt{13}} = 0.98$$

hence

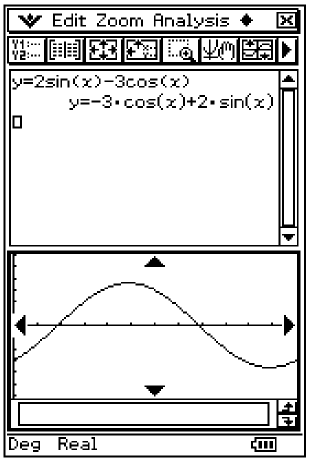
$$2\sin\theta - 3\cos\theta = \sqrt{13}\sin(\theta - 0.98)$$

9.

Ques 7



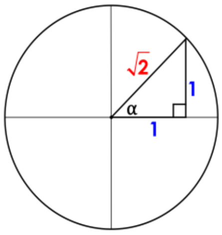
Ques 8



10.

$$a\cos(\theta-\alpha) = a(\cos\theta\cos\alpha + \sin\theta\sin\alpha)$$

(a) $\sqrt{1^2+1^2} = \sqrt{2}$



$$\alpha = \cos^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4}$$

hence

$$\cos\theta + \sin\theta = \sqrt{2}\cos\left(\theta - \frac{\pi}{4}\right)$$

(b) The maximum value of $\sqrt{2}\cos\left(\theta - \frac{\pi}{4}\right)$ is $\sqrt{2}$ (its amplitude) and occurs when

$$\cos\left(\theta - \frac{\pi}{4}\right) = 1$$

$$\theta - \frac{\pi}{4} = 0$$

$$\theta = \frac{\pi}{4}$$